# Tyngdpunkten 2026

Harder

November 2026

Each problem is worth at most 10 points. On problems with several parts, points can be awarded for a part even if the earlier parts have not been solved. Well-justified graphical solutions are always awarded full marks. Good luck!

**1. One-handed gym bro** Nico has been going to the gym a lot lately and has now started doing the military press (overhead press) with only his right arm, with his hand in the middle of the bar. On the bar there is already a 4 kg weight plate on the right side. He now wants to add another 5 kg weight plate on the left side of the bar. Assume that his hand can only push straight upwards on the bar and cannot twist it. The positions of the hand and the weight plates are shown in Figure 1. At what distance $x$ from his hand should he place the new weight plate for the bar to balance?[1]

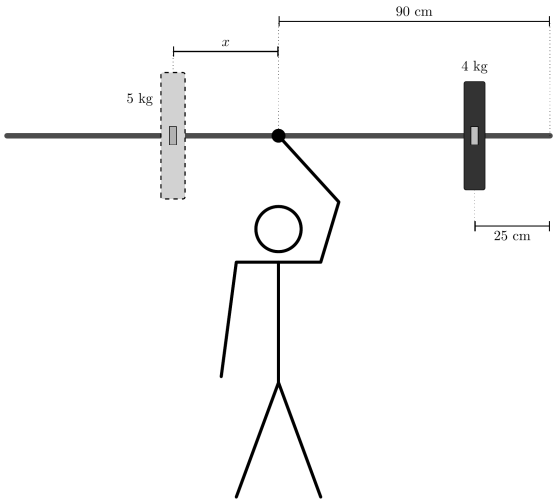



Figure 1: Nico's military press with only his right arm. The dashed weight plate is the new one. The figure is not to scale.

**2. Neutron + star = true** Normally, free neutrons decay according to the following reaction:

$$n \to p^+ + e^- + \bar{\nu}_e$$

In neutron stars the pressure is so high that this reaction cannot take place. Because of the very strong gravity, another reaction takes place instead, something called electron capture:

$$p^+ + e^- \to n + \nu_e$$

The masses of the proton, the neutron and the electron are $m_p = 1.007276$ u, $m_n = 1.008664$ u and $m_e = 0.00054858$ u. The mass of the neutrino is not known exactly, but lies somewhere between 0 and $5 \cdot 10^{-10}$ u.[2] Calculate the smallest possible speed that an electron must have for electron capture to be able to take place, given that the proton is at rest.

*Hint:* At these speeds, the relativistic formula for kinetic energy must be used:

$$E_k = \left( \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) mc^2.$$

[1] Nico is so strong that the weight is no problem. The hard part is keeping the bar balanced.

[2] For a long time it was believed that neutrinos have no mass. In 2015, Takaaki Kajita and Arthur B. McDonald received the Nobel Prize in Physics for showing that neutrinos can change between different flavours, something that is only possible if they have mass. Exactly how large the mass is is still not known.

**3. Doing doing doing** Two small bodies, both with mass $m$, can slide without friction along a fixed ring. Two springs, both with spring constant $k$, are threaded onto the ring so that they follow its shape. The springs are attached to the bodies but not to the ring. When the bodies are moved a little closer to each other, one spring is compressed while the other is stretched. When the bodies are then released, they start oscillating back and forth. What is the period of the oscillation? You do not need to take gravity into account.

**4. Eclipsing variable** Two stars move in circular orbits around their common centre of mass with constant angular velocity, and we are located exactly in the orbital plane.[3] The absolute surface temperatures of the stars are $T_1$ and $T_2$, where $T_1 > T_2$, and their radii are $R_1$ and $R_2$, where $R_1 > R_2$. Figure 2 shows the intensity $I$ from the star system measured from Earth as a function of time, expressed as a fraction of the largest measured intensity $I_{\max}$. Treat the stars as black bodies and assume that each star is seen from Earth as a uniformly bright disc. Determine the ratios $T_1/T_2$ and $R_1/R_2$.

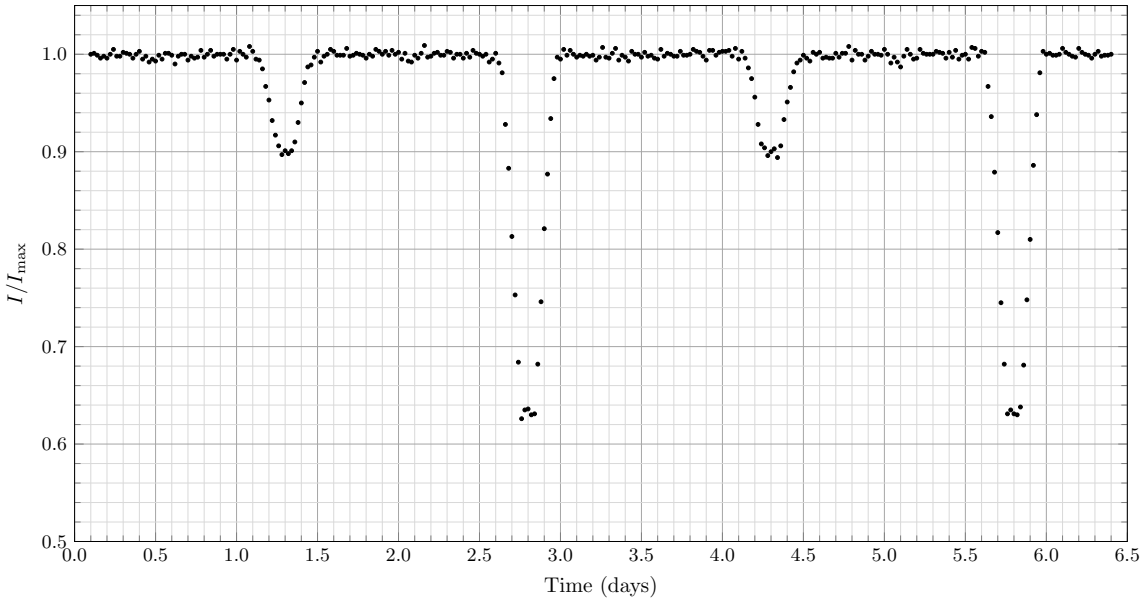



Figure 2: The measured relative intensity of the star system as a function of time.

**5. As all things should be** You have a circular, rigid bracelet with a bead that can slide without friction along the bracelet. The radius of the bracelet is $R$, the mass of the bead is $m$ and the bead is small compared to the bracelet. When the bracelet is held fixed in a vertical plane, you can balance the bead for a short while at the very top of the bracelet, but the slightest displacement makes the bead fall down. To solve this, you place a small ball with the positive charge $Q$ at the bottom of the bracelet and give the bead the positive charge $q$, see Figure 3. What is the minimum charge $q$ required for the bead to be in stable equilibrium at the very top of the bracelet?

*Hint:* You may need to use the approximations $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, which both hold for small angles $\theta$.

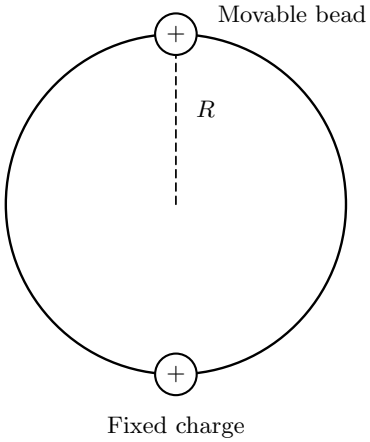
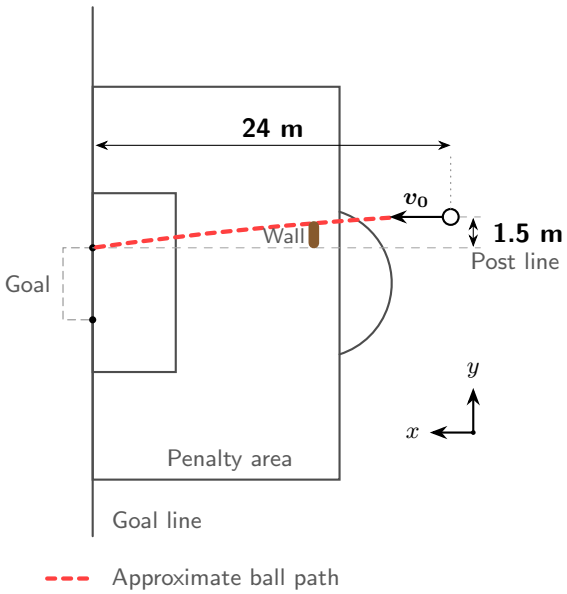



Figure 3: The bracelet with the bead at the very top and the fixed charge at the very bottom.

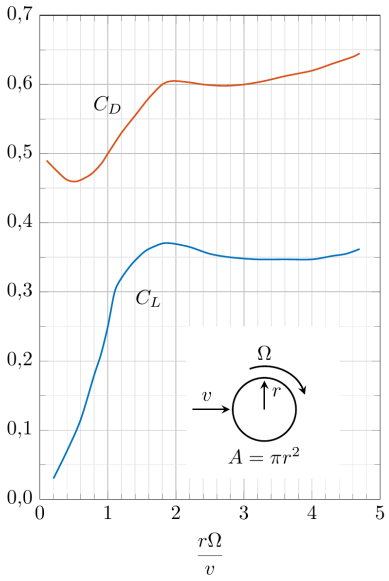
[3]The motion of the stars takes place entirely in a two-dimensional plane in space. This plane is called the *orbital plane*.

**6. And it's a gooooaaalll!!!** A ball that travels through the air while spinning is affected, in addition to air resistance, by a force perpendicular to the direction of motion, the so-called *Magnus force*. For a ball spinning about a vertical axis, as in a curled free kick, the Magnus force points sideways and makes the ball curve. The magnitudes of the air resistance and the Magnus force are determined by a drag coefficient $C_D$ and a lift coefficient $C_L$, respectively, which both depend on, among other things, the angular speed $\Omega$ (in rad/s). The relationship is shown in Figure 4b.
A free kick is shot from a distance of 24 metres with the speed $v_0 = 25$ m/s straight in the $x$-direction. To avoid the opposing team's wall, the ball is curled so that it curves 1.5 m to the left, and it goes in off the post.[4] A sketch of the goal is shown in Figure 4a. According to the official rules, a football should have a circumference of 69 cm and a mass of 430 g. The density of air is 1.3 kg/m$^3$. What angular speed did the ball have when it was kicked? Air resistance can be neglected.[5]

*Hint:* The magnitude of the Magnus force can be described by the formula $F_L = \frac{1}{2} C_L \rho A v^2$, where $\rho$ is the density of air, $A$ is the cross-sectional area of the ball and $v$ is the speed of the ball.




(a) Sketch of the free kick seen from above. The figure is not to scale.




(b) The drag coefficient $C_D$ and the lift coefficient $C_L$ for a spinning sphere as a function of $r\Omega/v$.

Figure 4: The free kick and the coefficients for a spinning sphere.

**7. Hose-Bella** Bella is watering her flowers with a garden hose. The hose has the inner radius $r_0$ and is pointed upwards at an angle $\alpha$ above the horizontal. Determine the radius of the water jet at the highest point of the jet. Neglect air resistance and assume that the water jet remains intact (does not break up into drops) throughout its flight.

[4] A similar goal was scored by Nathan Saliba in the match between Canada and Qatar in this year's World Cup, a very nice goal!
[5] This is a simplification. In reality the ball is noticeably slowed down on its way to the goal, so the answer is only an estimate.

**8. Young's double-slit experiment** In 1801, Thomas Young showed that light behaves like a wave. In the classic version of his experiment, light of a single wavelength passes through two narrow, parallel slits with slit width $a$ and slit separation $d$, see Figure 5. Figure 6 and Figure 7 show the result from such a setup.
If one of the slits is covered in the same setup, the light on the screen is instead distributed according to[6]

$$I(\theta) = I_0 \left( \frac{\sin \beta}{\beta} \right)^2, \qquad \beta = \frac{\pi a \sin \theta}{\lambda},$$

where $I_0$ is the intensity at the centre of the pattern. This gives intensity minima in the directions where

$$a \sin \theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \ldots$$

Determine the ratio $d/a$.

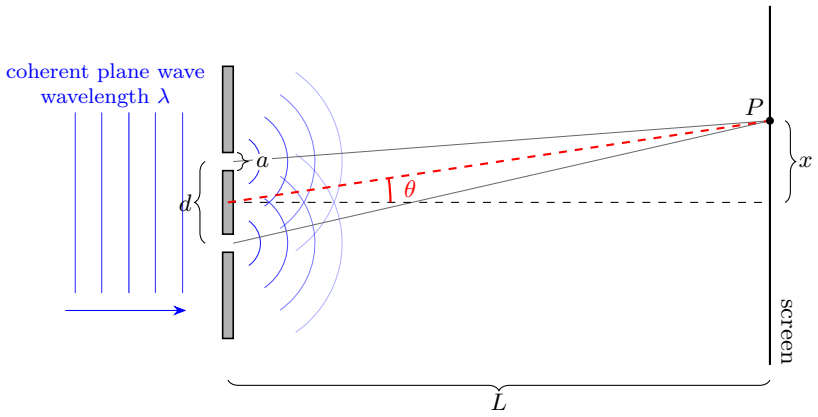



Figure 5: Young's double-slit experiment. A coherent plane wave with wavelength $\lambda$ is incident on two slits with width $a$, whose centre lines are at the distance $d$ from each other. The light from the slits reaches the point $P$ on the screen, which is at the distance $L$ from the slits. The point $P$ is at the distance $x$ from the centre of the pattern and is seen from the slits at the angle $\theta$. The figure is not to scale.

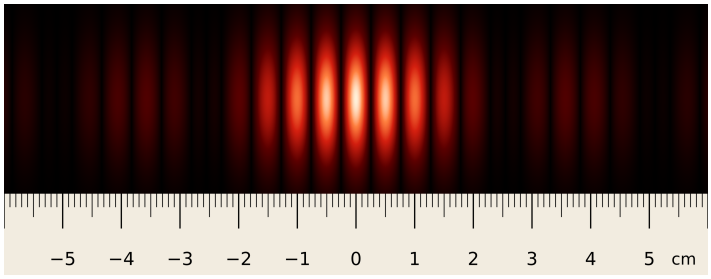



Figure 6: The light pattern on the screen. The scale of the ruler is in centimetres.

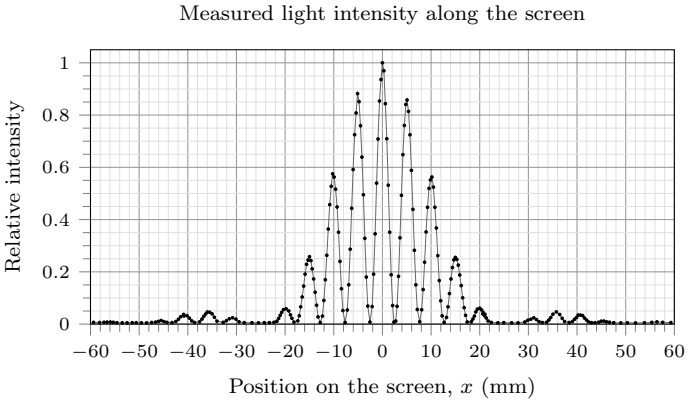



Figure 7: Measured relative light intensity as a function of the position $x$ on the screen, counted from the centre of the pattern.

[6]The expression may seem worrying, since it implies division by zero when $\beta = 0$. One can, however, show that $\lim_{\beta \to 0} \sin \beta / \beta = 1$.

**9. Maxwell's bracelet** A long solenoid (length $\ell \gg R_0$) with radius $R_0$ and turn density 0.5 turns/mm carries the current $I(t) = I_0 \sin(\omega t)$, where $I_0 = 2$ A and $\omega = 50$ rad/s. A cross-section of the solenoid is shown in Figure 8, with the current directions marked at an instant when the current is positive. A single-turn loop with resistance $R = 1\,\Omega$ is either wound around the solenoid (radius of the loop $R_l > R_0$, left image) or placed inside the solenoid ($R_l < R_0$, right image). The plane of the loop is perpendicular to the arrow, which indicates the longitudinal direction of the solenoid, and the centre of the loop coincides with the centre of the solenoid. Determine the maximum value of the induced current in both cases. Express the answers in terms of $R_0$ and $R_l$, respectively.

*Hint:* The magnitude of the magnetic field inside a long solenoid is given by $B = \mu_0 n I$, where $\mu_0$ is the magnetic constant, $n$ is the turn density and $I$ is the current through the solenoid. Outside the solenoid the magnetic field is zero.

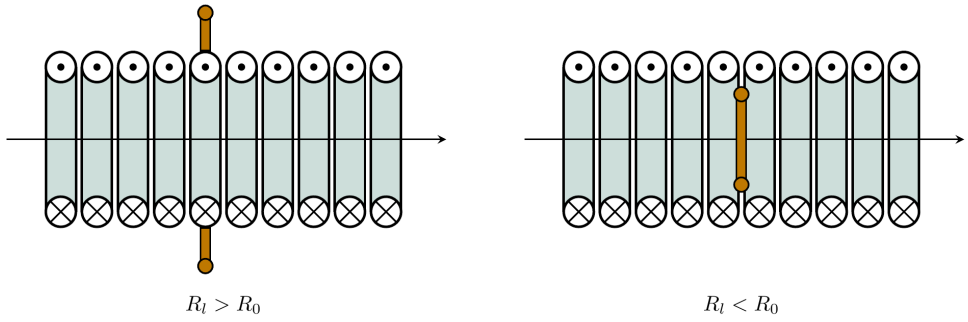



Figure 8: Cross-section of the solenoid with the loop (yellow) outside the solenoid (left) and inside the solenoid (right). The arrow indicates the longitudinal direction of the solenoid.

**10. Bowler golf** A bowler is trying to figure out how to play golf. She therefore drops a bowling ball and a golf ball together from the height $h$ above a flat surface. The bowling ball is much heavier than the golf ball, and both have radii that are much smaller than $h$. The bowling ball bounces off the surface and immediately afterwards collides with the golf ball. The balls are dropped so that all motion is vertical before the balls collide with each other, and the golf ball hits the bowling ball at the angle $\alpha$ from the highest point of the bowling ball, see Figure 9. All collisions are elastic and the friction between the balls can be neglected.
After the collision, the golf ball travels away from the bowling ball and lands at the distance $l$ from the point where the bowling ball hit the surface. Air resistance can be neglected. The height $h$ is fixed, but the angle $\alpha$ can be chosen freely. What is the largest distance $l$ that can be achieved, and for which angle $\alpha$ does this occur?

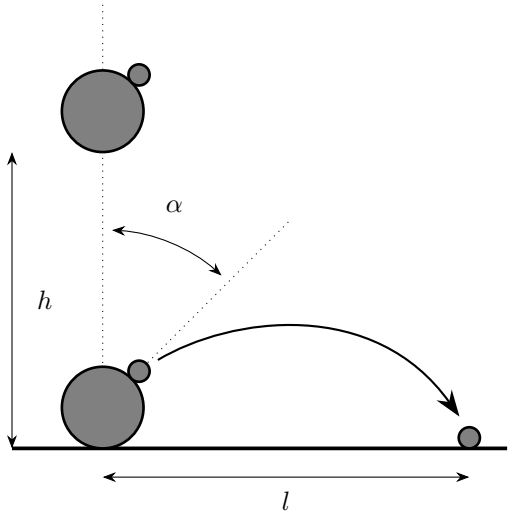
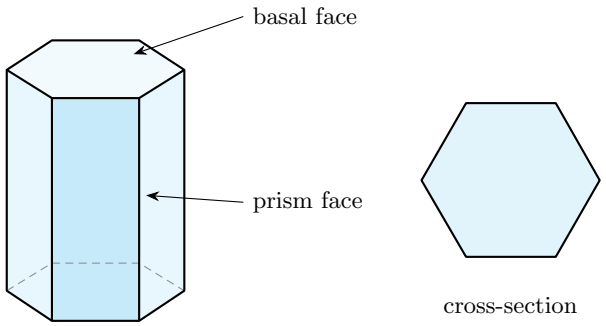



Figure 9: The setup in bowler golf, seen from the side.

**11. Winter's optical magic** On a cold winter day one can sometimes see bright rings around the Sun, so-called *halo rings*, see Figure 10a. The most common ones have angular radii of roughly 22° and 46°.[7] The rings arise when sunlight is refracted in small ice crystals in the air. The crystals have the shape of regular hexagonal prisms, see Figure 10b, and are randomly oriented. Ice has refractive index $n = 1.31$, air has refractive index 1 and the sunlight can be assumed to be parallel.



(a) Halo rings around the Sun.




(b) An ice crystal seen from the side and from above.

Figure 10: Halo rings and the ice crystals that cause them.

A ray that enters through one of the faces of the crystal and exits through another, without being reflected inside the crystal, is refracted in the same way as in a prism: two plane faces that form the angle $A$ with each other, see Figure 11.

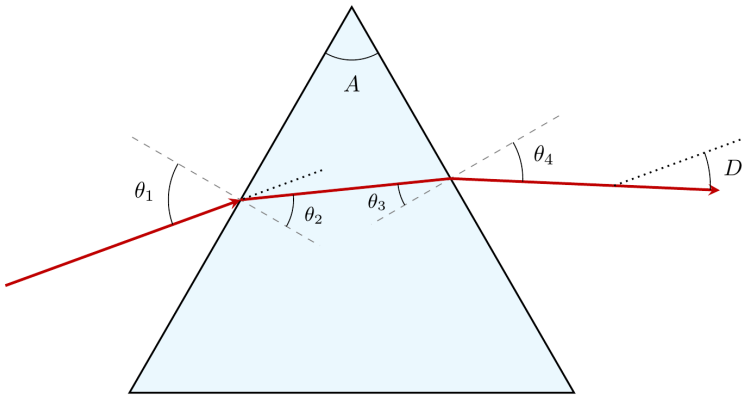



Figure 11: Ray path through a prism with the angle $A$ between the faces. The dashed lines are the face normals.

a) A ray of light is refracted into a prism with the angle of incidence $\theta_1$ and out with the angle $\theta_4$, as in Figure 11. Show that the deviation is

$$D = \theta_1 - A + \arcsin\Big(n \sin\big(A - \arcsin(\sin\theta_1/n)\big)\Big).$$

*Hint:* The *deviation* $D$ is the angle between the ray's original and final directions of travel, see Figure 11. You may assume that the ray path is as shown in the figure, and that the ray goes straight across the prism so that everything can be drawn in one plane. Rays that go obliquely through the prism do not affect the answer to the problem.

b) Since the crystals are randomly oriented, the light hits the crystals with many different angles of incidence. Explain why the light nevertheless gathers around a specific deviation, and determine this deviation expressed in terms of $A$ and $n$.

*Hint:* You may use without proof that $D$, as a function of $\theta_1$, has exactly one minimum.

c) What values can $A$ have for a ray that enters through one of the faces of the crystal and exits through another? Show that two of these give the rings at 22° and 46°, and explain why the others do not give any ring.

[7] The angular radius means the angle between the direction towards the Sun and the direction towards the ring, as seen by the observer.