HÖJDPUNKTEN 2026

Lower Secondary Competition, 20 March 2026

Solutions

Problem 1. We have started filling out a multiplication table for the three unknown positive integers a, b and c . Fill in the rest. *Only answer required*

	a	b	c
a		65	
b			
c	35		49

Solution. The solution is shown below and is obtained when $a = 5$, $b = 13$ and $c = 7$, or alternatively when $a = -5$, $b = -13$ and $c = -7$. One way to find it is by prime-factorising the numbers in the table. Teams that only stated the values of the variables received a point deduction.

	a	b	c
a	25	65	35
b	65	169	91
c	35	91	49

□

Problem 2. How many three digit numbers are there that neither start nor end with 99?

Solution. **Answer: 882**

There are 900 three-digit numbers (one hundred for each hundreds digit). Of these, 18 begin or end with 99, namely

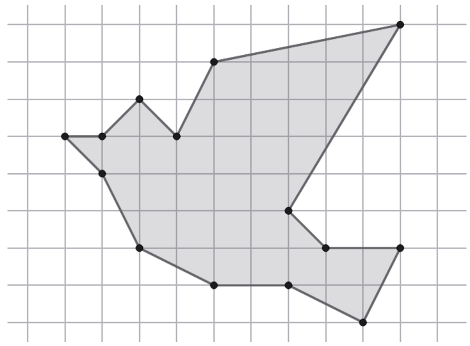
$$\begin{gathered}199,\ 299,\ 399,\ 499,\ 599,\ 699\\ 799,\ 899,\ 999,\ 990,\ 991,\ 992\\ 993,\ 994,\ 995,\ 996,\ 997,\ 998\end{gathered}$$

The number of three-digit numbers that neither begin nor end with 99 is therefore

$$900 - 18 = 882.$$

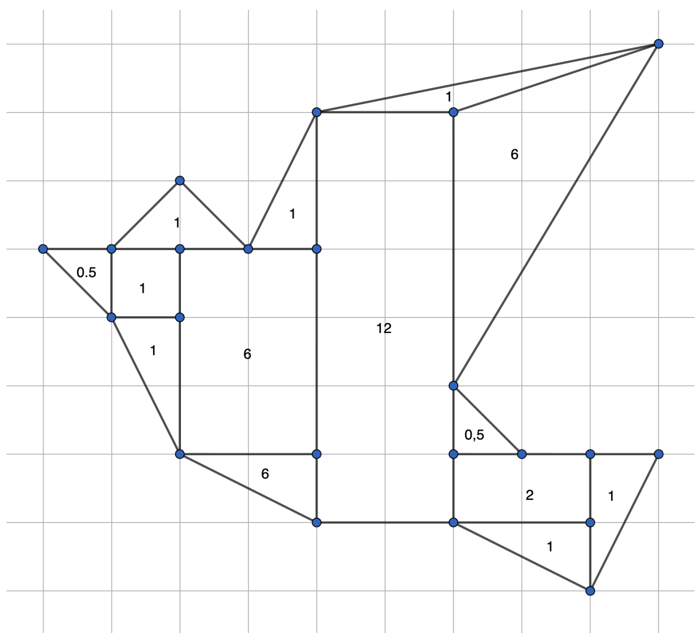
□

Problem 3. Calculate the area of the shaded region. Every square in the grid has area 1.



Solution. **Answer:** 35

We split the region into triangles and rectangles as in the picture below. The area of each part is easy to compute ($\frac{b \cdot h}{2}$ for triangles and $b \cdot h$ for rectangles). The area of the region is the sum of all these areas, which equals 35.



□

Problem 4. A scatterbrained time traveller got stuck in the year 1 A.D with only a broken time machine available. The time machine only has three working buttons, which do the following:

- $(+1)$ — Travel a year forward.
- (-1) — Travel a year backwards.
- $(\times 3)$ — Triple the current year.

The time machine only has energy left for ten button presses. Describe how the time traveller can make it back to the year 2026 A.D. *(Only answer required)*

Solution. **Solution**

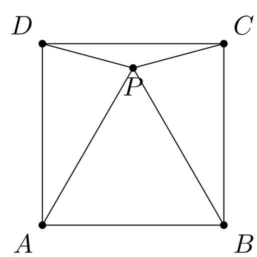
```
(x3) -> 3 A.D
(x3) -> 9 A.D
(-1) -> 8 A.D
(x3) -> 24 A.D
(+1) -> 25 A.D
(x3) -> 75 A.D
(x3) -> 225 A.D
(x3) -> 675 A.D
(x3) -> 2025 A.D
(+1) -> 2026 A.D
```

Alternatively: to get from 9 to 25 one can also press `x3-1-1`. □

Problem 5. Let A, B, C and D be the corners of a square in that order. Let P be a point inside the square such that $\triangle ABP$ is an equilateral triangle. How big is the angle $\angle CPD$?

Solution.

$$\boxed{\angle CPD = 150^\circ}$$



Since $ABCD$ is a square, we have

$$AB = BC = CD = DA,$$

and since $\triangle ABP$ is equilateral, we additionally have

$$AB = AP = BP \quad \text{and} \quad \angle APB = 60^\circ.$$

Now

$$\angle PAD = \angle BAD - \angle BAP = 90^\circ - 60^\circ = 30^\circ.$$

Since $AD = AB = AP$, the triangle APD is isosceles, so

$$\angle APD = \angle ADP = \frac{180^\circ - 30^\circ}{2} = 75^\circ.$$

Similarly,

$$\angle PBC = \angle ABC - \angle ABP = 90^\circ - 60^\circ = 30^\circ.$$

Since $BP = AB = BC$, the triangle BPC is isosceles, so

$$\angle BPC = \angle BCP = \frac{180^\circ - 30^\circ}{2} = 75^\circ.$$

The angles around the point P are therefore

$$\angle APB = 60^\circ, \qquad \angle BPC = 75^\circ, \qquad \angle DPA = 75^\circ.$$

Hence

$$\angle CPD = 360^\circ - 60^\circ - 75^\circ - 75^\circ = 150^\circ,$$

as we wanted to show. □

Problem 6. Ida has started a poetry club where members meet and write poems together. At every meeting, each member of the club writes one poem that they put in a joint poetry collection. To the first meeting, only Ida shows up, but more and more members join and no member ever stops showing up. However, there is no meeting where more than one new member joins. After eight meetings, the poetry collection consists of 22 poems, where 14 of them were written at the last four meetings. How many members does the poetry club have, and on which meetings did they join?

Solution. During the first four meetings, $22 - 14 = 8$ poems were written. Let n be the number of new members (including Ida) who showed up during this period. If $n = 2$, they could at most have written $4 + 3 = 7$ poems. If $n = 4$, that would mean they wrote exactly $4 + 3 + 2 + 1 = 10$ poems. The case $n = 3$, however, is possible if and only if one person joined on day 2 and another on day 4. These three members will write $3 \times 4 = 12$ poems during the remaining four days.

The remaining members (those who showed up during the last four days) thus wrote exactly two poems. If two people had shown up they would have needed to write at least three poems, so only one person joined, namely on day 7. In total the club has four members (including Ida). They joined on day 1 (Ida), day 2, day 4 and day 7. □

Problem 7. John has two lawn mowers named Alfons and Bosse that consume diesel at rates of $8 \frac{\mathrm{dL}}{\mathrm{h}}$ and $6 \frac{\mathrm{dL}}{\mathrm{h}}$, respectively. To compare the two, he first uses Alfons to cut half the lawn, and then Bosse to cut the other half. In total this took 15 minutes and consumed 1.7 dL of diesel. Which lawn mower consumed the least fuel?

Solution. Let x be the time (in minutes) for which Alfons was used. Then $15 - x$ is the total time Bosse was used. Computing the amount of diesel they must have consumed gives

$$\begin{aligned}
x\,\text{min} \cdot \frac{8\,\frac{\text{dL}}{\text{h}}}{60\,\frac{\text{min}}{\text{h}}} + (15-x)\,\text{min} \cdot \frac{6\,\frac{\text{dL}}{\text{h}}}{60\,\frac{\text{min}}{\text{h}}} &= 1.7\,\text{dL} \\
x \cdot \frac{8}{60} + (15-x) \cdot \frac{6}{60} &= 1.7 \\
8x + (15-x) \cdot 6 &= 60 \cdot 1.7 \\
x \cdot (8-6) &= 60 \cdot 1.7 - 15 \cdot 6 \\
x &= (60 \cdot 1.7 - 15 \cdot 6)/2 \\
&= (102 - 90)/2 \\
&= 6.
\end{aligned}$$

Therefore Alfons used $6 \cdot 8/60 = 0.8$ dL of diesel in total and Bosse used 0.9 dL in total. Since they each cut the same amount of lawn, Alfons is the more fuel-efficient mower. □

Problem 8. A grocery store has a sale: Buy at least four fruits and get the cheapest one for free. The first customer buys three oranges and one banana. The second one buys three oranges and two lemons. The third one buys two bananas and two lemons. It turns out that all three customers paid exactly 35 kr for their fruits. Fins the price for each individual fruit.

Solution. Let a, b and c denote the unit prices of oranges, bananas and lemons, respectively. If $c \geq b$, customer 2 would have paid more than customer 1, which they did not, so we must have $b > c$. If $a \geq b$, customer 2 would have paid more than customer 3, so we must have $b > a$. We conclude that the bananas are the most expensive of the three fruits. It remains to determine whether a or c is larger. Customers 1 and 3 pay $2a + b$ and $b + 2c$ respectively for their fruit. If $c > a$, then

$$35 = 2a + b < 2c + b < b + 2c = 35,$$

which is impossible. Hence $a \geq c$. We have now determined the price ordering of the fruits, so we know how many fruits of each kind each customer must pay for. This gives the system of equations

$$\begin{cases} 2a + b = 35 & (1) \\ 3a + c = 35 & (2) \\ 2b + c = 35 & (3) \end{cases}$$

From (1) and (2) we get $b = 35 - 2a$ and $c = 35 - 3a$. Substituting into (3) gives

$$\begin{aligned}
2(35-2a) + (35-3a) &= 35 \\
\iff 105 - 7a &= 35 \\
\iff a &= \frac{105-35}{7} = 10.
\end{aligned}$$

Hence $b = 35 - 20 = 15$ and $c = 35 - 30 = 5$.

Answer: The oranges cost 10 kr, the bananas cost 15 kr and the lemons cost 5 kr. □

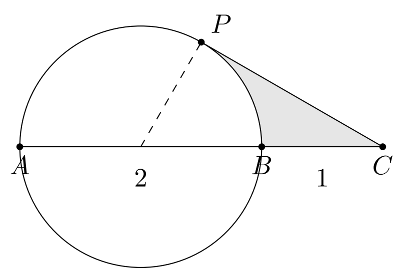
Problem 9. Jakob draws 4 points in the plane such that no three lie on a line. Choose one of these and draw two lines from it to two other points. This creates an angle (we choose the one smaller than 180°). What is the result if we sum all such angles?

Solution. Let A, B, C and D denote the four points. There are exactly four ways to choose the first point. Once the first point has been chosen, there are three ways to choose the two other points. So there are $3 \times 4 = 12$ angles to sum, namely $\angle ABC$, $\angle BCA$, $\angle CAB$, $\angle ACD$, $\angle CDA$, $\angle DAC$, $\angle ABD$, $\angle BDA$, $\angle DAB$, $\angle BCD$, $\angle CDB$, and $\angle DBC$. We see that these are the angles of four triangles ($\triangle ABC$, $\triangle ACD$, $\triangle ABD$ and $\triangle BCD$). The angle sum of a triangle is always 180°, so the sum of all these angles is $4 \cdot 180° = 720°$. □

Problem 10. The points A, B and C lie on a line in that order such that $|AB| = 2$ and $|BC| = 1$. The point P lies on the circle with diameter AB such that the line PC is tangent to it. Find the area of the region enclosed by the segments BC and CP together with the circular arc between P and B on the circle with diameter AB.

Solution.

Solution.



Let O be the midpoint of AB. Then $OA = OB = 1$, and since $|BC| = 1$ we have $OC = 2$.

Since PC is tangent, $OP \perp PC$, so in the right triangle OPC:

$$PC = \sqrt{OC^2 - OP^2} = \sqrt{4 - 1} = \sqrt{3}.$$

Furthermore,

$$\cos \angle POC = \frac{OP}{OC} = \frac{1}{2} \Rightarrow \angle BOP = \angle POC = \frac{\pi}{3}.$$

The area of triangle BCP is

$$[\triangle BCP] = \frac{1}{2} \cdot BC \cdot \text{height} = \frac{1}{2} \cdot 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}.$$

The circular segment between the arc BP and the chord BP is

$$\text{sector } BOP - \triangle BOP = \frac{\pi}{6} - \frac{\sqrt{3}}{4}.$$

The desired area is therefore

$$\frac{\sqrt{3}}{4} - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}.$$

$$\boxed{\frac{\sqrt{3}}{2} - \frac{\pi}{6}}$$

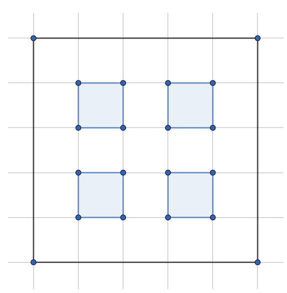
□

Problem 11. A square carpet is sewn together from square pieces of fabric of which half are large and half are small. The large pieces of fabric are 2×2 dm and the small ones are 1×1 dm. The pieces are sewn together without any overlap. What is the smallest possible size of this carpet?

Solution. Let n be the number of pieces of each kind and s the side length of the carpet. Then

$$5n = s^2,$$

so s is divisible by 5. We first check whether $s = 5$ works. This gives $n = 5^2/5 = 5$, but if we try to place the large pieces on a 5×5 dm carpet we see that each large piece must cover exactly one of the squares in the picture.



There are only four such squares, so we can fit at most 4 large pieces, which is not enough. Hence $s = 5$ does not work.

The next number to check is $s = 10$, and we see that it works. Divide a 10×10 grid into 25 large squares. Choose five of these squares and divide each of them into four small squares. This gives 20 small squares and 20 large squares, as desired.

Answer: The carpet is at least 1×1 square metre. □

Problem 12. Find all solutions to the system of equations

$$\begin{cases} p + q + r = s \\ p + 2q + 3r = 5t \end{cases}$$

where p, q, r, s and t are prime numbers.

Solution. Subtracting the first equation from the second, we get

$$q + 2r = 5t - s.$$

Both s and t are obviously greater than 2, hence odd, so the right-hand side is odd $-$ odd $=$ even. This means q is even, i.e. $q = 2$.

From the first equation we now see that $p + r$ must be odd. This gives two cases:

Case 1 $p = 2$ and r odd: The second equation becomes

$$2 + 4 + 3r = 5t.$$

But the left-hand side is a multiple of 3, so $t = 3$, and hence $r = (15-4-2)/3 = 3$. The first equation then gives $s = 7$. Substituting back, we see that this solves the system.

Case 2 p odd and $r = 2$: The second equation now becomes

$$p + 4 + 6 = 5t \iff p = 5t - 10.$$

The right-hand side is now a multiple of 5, so $p = 5$ and $t = 3$. But substituting this into the first equation gives $s = 5 + 2 + 2 = 9$, which is not prime.

Answer: The system has only one solution, namely $(p, q, r, s, t) = (2, 2, 3, 7, 3)$.

□