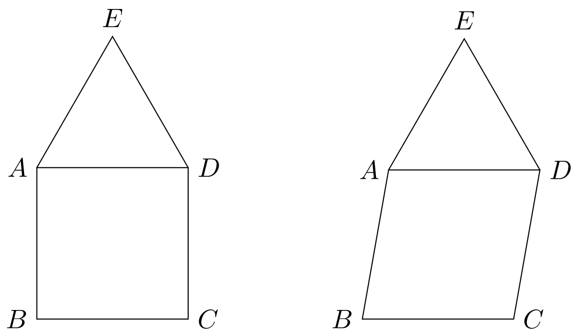
HÖJDPUNKTEN 2026

Open competition 20-21 March 2026

Time allowed: 5 hours
Allowed aids: pen, eraser, compass and ruler only
Each problem is worth 7 points. Full credit requires justification unless otherwise stated.
Answer only one problem per submitted page, and write your team name on each page.

Problem 1. A grocery store has a sale: Buy at least four fruits and get the cheapest one for free. The first customer buys three oranges and one banana. The second one buys three oranges and two lemons. The third one buys two bananas and two lemons. It turns out that all three customers paid exactly 35 kr for their fruits. Fins the price for each individual fruit.

Problem 2. The picture shows a two dimensional model of a house consisting of six line segments of equal length. A strong gust of wind has made it so that the walls of the house (AB and CD) have started to tilt (by an angle smaller than $30°$). Prove that the angle $\angle BEC$ does not depend on the tilt and calculate the size of this angle.



Problem 3. Find all solutions to the system of equations

$$\begin{cases} p + q + r = s \\ p + 2q + 3r = 5t \end{cases}$$

where p, q, r, s and t are prime numbers.

Problem 4. Let n be a positive integer. In the old castle, the royal glazier is working to install a new window in the throne room. She must place $\frac{1}{2}n(n+1)$ purple glass panes and $\frac{1}{2}n(n-1)$ yellow glass panes in a $n \times n$ grid of panes such that no two rows have the same number of purple panes and no two columns have the same number of purple panes. In how many ways can she do this?

Problem 5. Let $\triangle ABC$ be a right-angled triangle with $\angle ABC = 90°$ and $|AB| < |BC|$. Let D be a point on the hypotenuse AC such that $|AB| = |BD|$. The point T lies on side BC and is such that $\angle ATB = \angle CTD$. Prove that the line through D perpendicular to BD splits the segment CT in half.

Problem 6. A colouring of the cells of a $n \times n$ grid in the colours red and blue is called *elegant* if it is possible to move between each pair of red cells, by moving between cells with a common side, without having to visit any third red cell. Let $R(n)$ be the largest possible number of red cells that an *elegant* colouring of a $n \times n$ grid can have. Determine

$$\lim_{n \to \infty} \frac{R(n)}{n^2}.$$

Problem 7. In Numberland, each city is named after a positive integer. Two citizens can only exchange letters with each other if they live in different cities whose names differ by a power of two (note that two citizens living in the same city cannot exchange letters). Given that Numberland has n citizens, what is the largest possible number of pairs of citizens in Numberland that can exchange letters with each other?

Problem 8. Matilda has an odd prime p and a grid with n rows, where n is a positive integer. On the k'th row ($1 \leq k \leq n$) she enters the digits of the number kn in base p, with the i'th digit (from the right) of the number being entered into column i for all i.

It turns out that if Matilda chooses any column of the grid and sums all digits entered into that column, the result will always be divisible by p. Prove that p divides n.

Problem 9. Ulrich wishes to synchronize his n clocks which have all stopped, where n is an odd positive integer. All of Ulrich's clocks have a 12-hour clock face with an hour hand and a minute hand. Ulrich can adjust each clock's minute hand directly at a speed of 10 revolutions per minute. In the worst case, how long must Ulrich spend adjusting minute hands if he chooses the time to which he synchronizes his clocks to optimally?

Problem 10. Let $\triangle ABC$ be an acute triangle with orthocenter H. Let Γ be the circle passing through H which is tangent to the circumcircle of triangle $\triangle ABC$ at A. Let M be the center of Γ. Suppose that Γ intersects lines BH and CH again at points D and E, respectively.

Prove that the circumcircle of triangle $\triangle MDE$ is tangent to line BC.

Problem 11. Scott has an infinite sheet of paper with a grid of equilateral triangles. He cuts out a connected figure by only cutting along the gridlines. Then, he folds the figure in the following way: each fold occurs along a gridline and results in the area of the shape being halved. After a finite number of folds, he is left with a triangle cell. Prove that what was left of the sheet of paper after he cut out his figure is connected.

[*By a connected part of the grid we mean a set of triangle cells such that it is possible to move between each pair of cells in this set by only moving between cells in the set with common sides.*]

Problem 12. An infinite sequence $r_0, r_1, r_2, \ldots$, consists of rational numbers, and for each integer $n \geq 1$ it holds that r_n is a root of the polynomial

$$x^n + r_{n-1}x^{n-1} + \cdots + r_0.$$

Prove that there exists a number N such that $r_n = r_N$ for all $n > N$.