HÖJDPUNKTEN 2026

Gymnasietävling - 20 March 2026

Time allowed: 4 hours
Allowed aids: pen, eraser, compass and ruler only
Each problem is worth 7 points. Full credit requires justification unless otherwise stated.
Answer only one problem per submitted page, and write your team name on each page.

Problem 1. A scatterbrained time traveller got stuck in the year 1 A.D with only a broken time machine available. The time machine only has three working buttons, which do the following:

- $(+1)$ — Travel a year forward.
- (-1) — Travel a year backwards.
- $(\times 3)$ — Triple the current year.

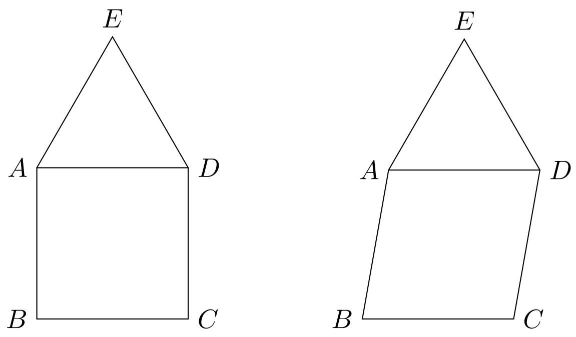
The time machine only has energy left for ten button presses. Describe how the time traveller can make it back to the year 2026 A.D. *(Only answer required)*

Problem 2. John has two lawn mowers named Alfons and Bosse that consume diesel at rates of $8\frac{\text{dL}}{\text{h}}$ and $6\frac{\text{dL}}{\text{h}}$, respectively. To compare the two, he first uses Alfons to cut half the lawn, and then Bosse to cut the other half. In total this took 15 minutes and consumed 1.7 dL of diesel. Which lawn mower consumed the least fuel?

Problem 3. A grocery store has a sale: Buy at least four fruits and get the cheapest one for free. The first customer buys three oranges and one banana. The second one buys three oranges and two lemons. The third one buys two bananas and two lemons. It turns out that all three customers paid exactly 35 kr for their fruits. Fins the price for each individual fruit.

Problem 4. Styrbjörn walks around in a grid of size $n \times m$, where $n \geq 2$ and $m \geq 2$ are integers. He starts in one of the corners, and every minute he walks into a square next to (i.e. sharing a side with) the one he is in. Styrbjörn is a bit restless, and therefore he does not want to visit the same square twice. He also does not want to walk two steps in a row in the same direction. For which values of n and m can Styrbjörn visit all $n \cdot m$ squares?

Problem 5. The picture shows a two dimensional model of a house consisting of six line segments of equal length. A strong gust of wind has made it so that the walls of the house (AB and CD) have started to tilt (by an angle smaller than 30°). Prove that the angle $\angle BEC$ does not depend on the tilt and calculate the size of this angle.



Problem 6. In the old castle, the royal glazier works with creating a new window for the throne room. She has to place ten purple panes and six yellow panes in a 4×4 grid such that no two rows and no two columns have the same number of yellow panes. In how many ways can she do this?

Problem 7. A square carpet is sewn together from square pieces of fabric of which half are large and half are small. The large pieces of fabric are 2×2 dm and the small ones are 1×1 dm. The pieces are sewn together without any overlap. What is the smallest possible size of this carpet?

Problem 8. Find all solutions to the system of equations

$$\begin{cases} p+q+r=s \\ p+2q+3r=5t \end{cases}$$

where p, q, r, s and t are prime numbers.

Problem 9. Consider $n \geq 3$ points in the plane where no three lie on a line. Choose one of the points and draw two lines from it to two other points. This creates an angle (we choose the one smaller than $180°$). What is the result if we sum all such angles?

Problem 10. Let $\triangle ABC$ be a right-angled triangle with $\angle ABC = 90°$ and $|AB| < |BC|$. Let D be a point on the hypotenuse AC such that $|AB| = |BD|$. The point T lies on side BC and is such that $\angle ATB = \angle CTD$. Prove that the line through D perpendicular to BD splits the segment CT in half.

Problem 11. Find all functions f, from the real numbers to the real numbers, such that the equality

$$f(f(x)+f(y)+yf(f(x))) = yf(f(x))+xf(x)$$

holds for all real numbers x and y.

Problem 12. Let a be a positive integer. Let a_1, a_2, a_3, $\ldots$, be the infinite sequence of positive integers satisfying $a_1 = a$ and $a_{n+1} = a_n + \gcd(a_n, n)$ for all positive integers n.

(a) Find all values of a for which $a_n \leq n + 2026$ for all positive integers n.

(b) For each positive integer a, prove that there exists a constant C such that $a_n \leq Cn$ for all positive integers n.

[*The greatest common divisor,* $\gcd(x, y)$*, of two integers* x *and* y *is the largest positive integer thet divides both* x *and* y*.*]