Tyngdpunkten 2025

Harder, English

November 2025

Each problem can give at most 10 points. Well-motivied graphical solutions always give maximal points. Good luck!

1. The Force In a galaxy far, far away, there is a planet called Dagobah. The planet is moving in a circular orbit around the star Darlo (the star is very massive, so you can assume it to be stationary). The gravitational force between the planet and the star has the magnitude $6.2 \times 10^{22}\,\mathrm{N}$, and the mass of the planet is $8.5 \times 10^{24}\,\mathrm{kg}$. It takes Dagobah 341 days to complete one full orbit around Darlo. Determine the distance between the planet and the star.

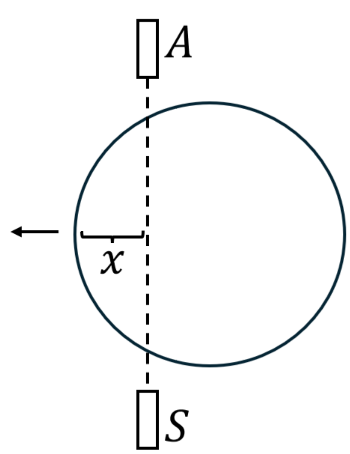
2. Raindrops Two spherical raindrops fall from a rain cloud. For one of the raindrops it takes 7 minutes to reach the ground and for the other it takes 4 minutes. Determine the ration between the radii of the raindrops. Assume that the density of air is constant and that the water drops reach terminal velocity almost immediately.

Hint: The air resistance on a falling body is an upward force, whose magnitude is given by $F = \frac{1}{2}\rho v^2 C_D A$, where ρ is the air density, A is the crossectional area of the body, v is speed of the body and C_D is a constant (that is the same for both raindrops).

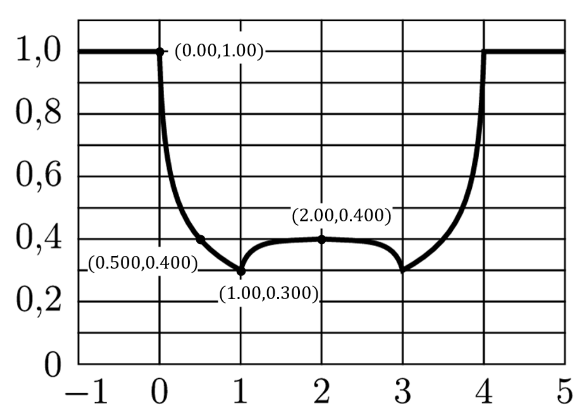
3. Rubidium Measurements indicate that 27.83% of all rubidium atoms currently on Earth are the radioactive ^{87}Rb isotope. The rest are the stable ^{85}Rb isotope. The half-life of ^{87}Rb is 4.75×10^{10} y, and it decays to strontium, which is stable. Assuming that no rubidium atoms have been formed since, what percentage of rubidium atoms were ^{87}Rb when our solar system was formed 4.6×10^{9} y ago?

4. Bobbing wooden cylinder A cylindrical glass with radius R is half-filled with water. In the glass there is a homogeneous cylindrical piece of wood with radius $r < R$ and mass m, whose base is always parallel to the water surface. The piece of wood has a lower density than water and therefore floats on the surface. The wood is pushed down and then released, so that it starts to bob up and down. Determine the period of this motion as a function of R, r, m, and the density ρ of the water. You may assume that some part of the wood is always above the water surface. Neglect surface tension, frictional losses, and any waves on the water surface.

5. Criminology An X-ray device (A) emits rays that are detected by a sensor (S). Between the X-ray device and the sensor, a cylinder with *thick* walls is placed (in the figure, only the outline of the cylinder is shown). The intensity of the X-ray beam measured by the sensor depends on the position x, as shown in the graph. Use Figure 1 to determine whether the cylinder contains any (illegal!) material that absorbs X-rays!



(a) A schematic illustration of the setup.



(b) The measured intensity as a function of the position x. Some points have been added for clarity, and the graph is completely symmetric around $x = 2$.

Figure 1: Figures for the problem.

6. A cylinder with a hole A uniform cylinder of radius a and mass 50 kg is placed on a horizontal surface. A cylinder hole is drilled through the cylinder with its center at a distance of $\frac{2a}{5}$ from the center of the original cylinder (cf. Figure 2). The axis of the hole is parallel to the axis of the original cylinder, and their centra are at the same height above the surface. After the hole is drilled, the mass of the cylinder is reduced to 43 kg. To keep the cylinder at rest, a horizontal force is applied to the top of the cylinder. Determined the magnitude and the direction of this force.

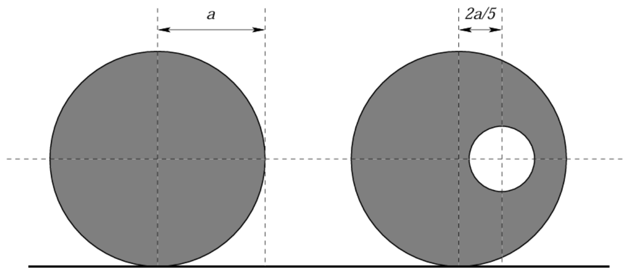


Figure 2: The setup.

7. Lifeguard Henrik is standing at point A on a straight beach, at a distance $l = 20\,\text{m}$ from the water. He sees his friend at point B in the water, a distance $L = 30\,\text{m}$ from the shore and also a distance $d = 80\,\text{m}$ from Henrik along the beach. Henrik wants to reach his friend as quickly as possible to give him a compliment. He can run on land with speed $v_l = 5.0\,\text{m/s}$ and swim with speed $v_s = 2.0\,\text{m/s}$. How should he plan his route to reach his friend as fast as possible? How long will it take? Assume that Henrik begins swimming as soon as he reaches the water. See Figure 3.

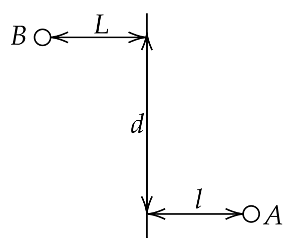


Figure 3: Illustration of Henrik and his friend.

8. Circuit with Charged Spheres Study the circuit in Figure 4, consisting of a voltage source with EMF $\mathcal{E}$ and two series-connected resistors, each with resistance R. To points 1, 2, and 3, metal spheres with radii r, ρ, and r, respectively, are connected via long, thin wires. Determine the charge on the spheres after they have been connected (and a long time has passed), assuming the spheres were initially uncharged. Assume that the charge on the circuit itself and on the wires is negligibly small, and that the internal resistance of the voltage source is 0. Also assume that the distances between the spheres are much larger than the radii of the spheres (which implies that one sphere essentially does not affect the potential of another sphere).

Hint: The electric potential of a metal sphere with charge q and radius r (relative to a point very far away) is given by:

$$V = \frac{q}{4\pi\epsilon_0 r},$$

where ϵ_0 is a constant.

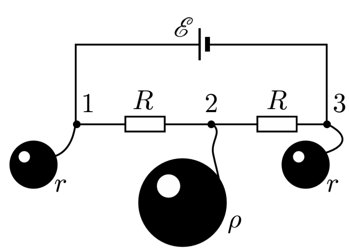


Figure 4: An illustration of the setup.

9. Ice and water You mix water of mass m_w with ice of mass m_{ice} in a container. The diagram shows the final state of the mixture as a function of the initial temperature T_w of the water and the initial temperature T_{ice} of the ice. For points below the curve in the diagram, all of the water ends up as ice, while for points above the curve the final state is a mixture of ice and liquid water.

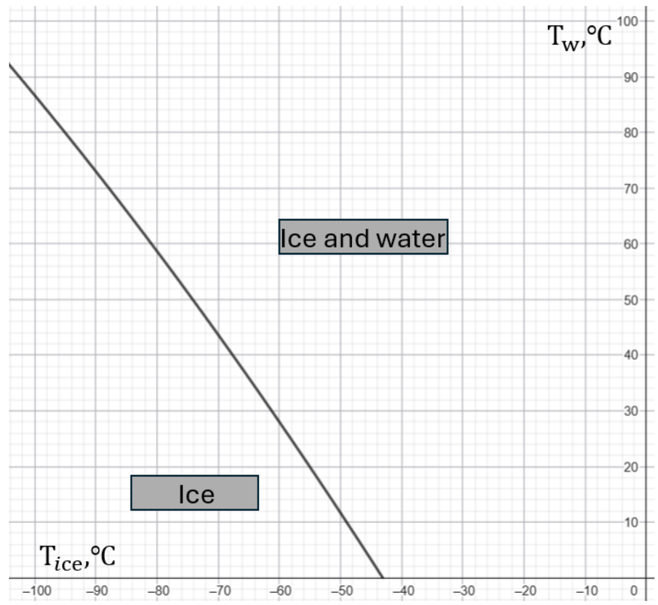


Figure 5: Final state when mixing ice and water.

The specific heat capacity of ice in the temperature range considered is $c_{\text{ice}}(T_{\text{ice}}) = c + \alpha T_{\text{ice}}$, where $c = 2100\,\text{J}/(\text{kg}^{\circ}\text{C})$, T_{ice} is the temperature of the ice in degrees Celsius, and α is a constant. The specific heat capacity of water is $2c$, and the latent heat (enthalpy) of fusion of water is $\lambda = 2cT$, where $T = 80\,^{\circ}\text{C}$. Use Figure 5 to determine α and $\gamma = m_{\text{ice}}/m_w$. Ignore heating of the container and all heat losses.

Remark: You can receive partial credit on this problem if you assume $\alpha = 0$ and compute γ based on that assumption. In that case you obtain an approximate value of γ.

10. Solar sail A reflecting solar sail with area A is located at a distance x from the Sun. The solar sail is perpendicular to the rays of the sun. If the Sun radiates the total power $P_{\odot}$, what is the force on the solar sail from the solar radiation?

11. (H)oops! A rigid ring with radius r stands vertically with two beads at the top, as shown in Figure 6. The beads can slide down the ring. The ring has mass m and each bead has mass m_b. The beads begin to slide down the ring in opposite directions with negligible initial velocity. The ring remains vertical throughout the entire motion. Determine the maximum value of $\frac{m_b}{m}$ such that the ring never leaves the ground during the beads' descent. Friction is negligible.

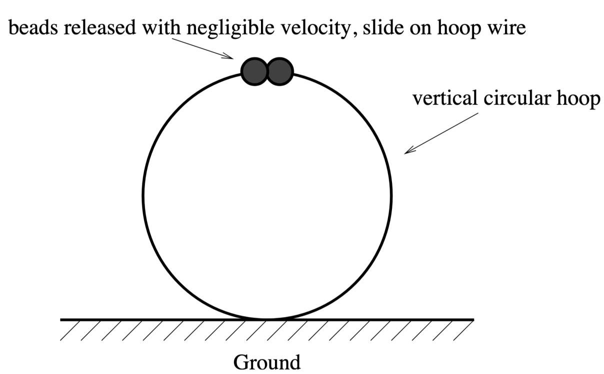


Figure 6: Hoop with beads.