Högstadietävling — 16 May 2025

Time allowed: 3 hours
Allowed aids: pen, eraser, compass, and ruler only
Each problem is worth 7 points.
Full credit requires justification unless otherwise stated.

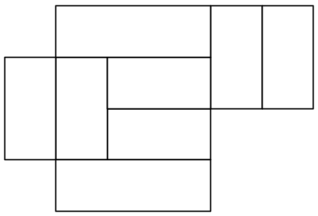
Problem 1. Write the numbers $1, 2, \ldots, 21$ in a circle in some order so that the difference between any two adjacent numbers is always either 8 or 13. *(Answer only required.)*

Problem 2. Over Friday, Saturday, and Sunday John ate some grapes. He ate the same number of red grapes each day, and in total over the three days he ate as many red grapes as green grapes. If John ate 5 grapes on Friday and 18 grapes on Saturday, how many *green* grapes did he eat on Sunday?

Problem 3. This year Höjdpunkten has a logo made of twelve squares that together form a rectangle (see the top of the page). If the height of this rectangle is 11 units, how wide is it?

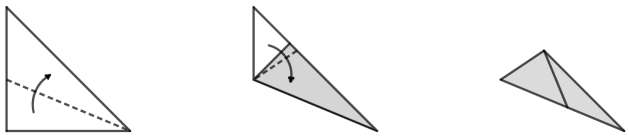
Problem 4. Sofia has a balance scale and nine weights of $1, 2, 4, 8, 16, 32, 64, 128$, and 141 grams. How can she balance all the weights on the scale with none left over? *(Answer only required.)*

Problem 5. We call two numbers *difference-divisible* if both are divisible by their difference. Place the numbers 1 through 8 in the figure below so that numbers in adjacent rectangles are difference-divisible. *(Answer only required.)*



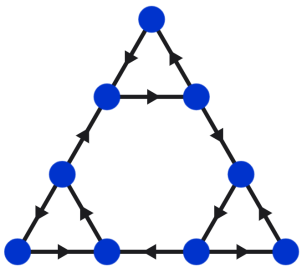
For example, 10 *and* 20 *are difference-divisible because* $20 - 10 = 10$ *divides both numbers. By contrast,* 17 *and* 25 *are not difference-divisible because neither is divisible by* $25 - 17 = 8$.

Problem 6. Theodor cuts a square sheet of paper in half along a diagonal to make a triangle. He then folds the paper twice as shown in the picture so that it becomes a smaller triangle. Determine all three angles of the new triangle.

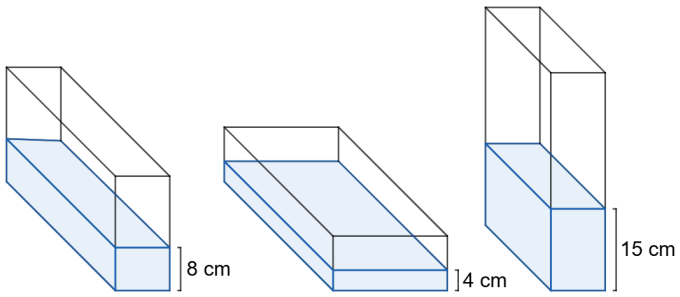


Problem 7. Emil has six stones whose weights are $1, 2, 3, 4, 5$, and 6 kilograms. Yesterday he put a label showing its weight on each stone, but he worries that his friend Ivar played a prank overnight and swapped the labels. Emil wants to decide whether the labels are correct by performing a number of weighings on his balance scale. After each weighing he learns which pan is heavier, or that they have the same weight. Can you suggest weighings Emil can perform that are guaranteed to expose Ivar if he moved the labels?
The fewer weighings your solution uses in the worst case, the more points you receive!

Problem 8. In a trivia game the board looks as in the figure below. You start on some square. Each round you move one step along an arrow, in the direction the arrow points. Is it possible to return to the square you started on after exactly 20 moves?

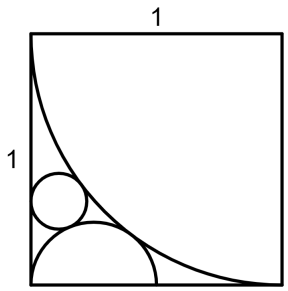


Problem 9. Cecilia pours three litres of water into a rectangular glass container with a lid. When she places it on a table, the water depth can be 8 cm, 4 cm, or 15 cm depending on how she rotates it. What is the volume of the container?



Problem 10. A *palindromic number* reads the same forwards and backwards, for example 494. If you multiply together all three-digit palindromic numbers, how many zeros does the product end with?

Problem 11. A quarter-circle, a semicircle, and a full circle are packed into a square of side length 1 unit, as in the figure below. Determine the diameter of a) the semicircle b) the full circle.



Problem 12. A frog is at the point $(0, 0)$ in the plane and starts jumping. Its first jump has length 1, and each subsequent jump is twice as long as the previous one. Every jump is made parallel to one of the coordinate axes. Which points can the frog reach by jumping in this way?