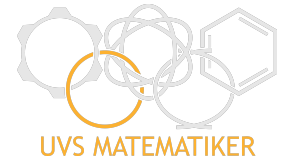


HÖJDPUNKTEN 2024

Högstadietävling, 8-9th of May 2024



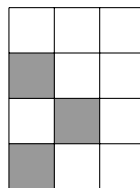
Time: 3 hours

Tools: Only pen, eraser, compass and ruler

Justify all your solutions. Only answers are not sufficient unless otherwise stated.

Problem 1. A prime number is a number greater than 1 that can only be divided by 1 and itself.

- (a) What are the 9 first prime numbers? (*Answer only required*)
- (b) Fill in the 9 first prime numbers in the unshaded boxes in the figure below so that the sum in each row and column ends with a 0. Each number must be used exactly once. (*Answer only required*)



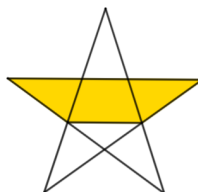
Problem 1

Problem 2. Julia has figured out a way to never run out of juice! Every time she has drunk half of her glass, she refills with water so that it is full again. That way it never ends. The problem is that the juice gets a little weaker each time. Julia started by 1 scoops of concentrated juice and 5 scoops of water in a jug. After that, she has drunk half the glass and refilled it with water three times.

- (a) What is the proportion of concentrated juice in the glass now?
- (b) Julia would like the juice to taste good again. She therefore intends to drink some of the way too weak juice, and then refill with more concentrate. How much of the glass must she drink in order for the concentration to be the same as it was initially?

Problem 3. The principal at your school wanted to count how many students there was in the school yard. She knows that there are at most 300 students, because that is how many people attend the school in total. Suddenly she got an idea! She told the students to organize themselves into groups of 5, and they did with no students left over. Then she asked all the students to stand in groups of 9 and it turned out to be 7 students left over. Finally, she asked everyone to stand in groups of 4, and once again, there were no students left over! How many students was there in the schoolyard?

Problem 4. The pentagon in the center is regular. What proportion of the star is colored?



Problem 4

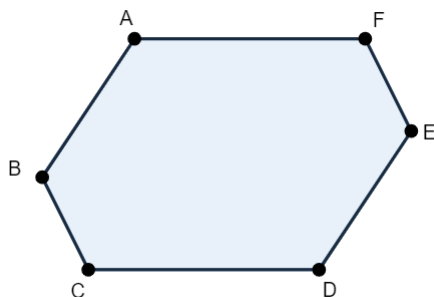
Problem 5. Among the numbers 1 through 12, there are in total 5 ones. How many ones are the among the numbers 1 through 2024?

Problem 6. At exactly midnight on Friday the 13th, the mad professor Sonja creates a slime monster weighing 1 kilogram. It grows 12 times its size in an hour, and at 1:00 am it splits into 12 smaller slimes. All of these then grow 24 ($= 2 \cdot 12$) times as large, and at 2:00 they split into 11 smaller slimes each. Their offspring in turn grow 36 ($= 3 \cdot 12$) times as large, and at 3:00 am they split into 10 smaller slimes, and so on. This reproduction continues until 12:00. Show that each slime then weighs more than a billion tons.

Problem 7. Tilda has a rectangular piece of cardboard that is 9 meters wide and 4 meters long. How can she cut it into two pieces which can be reassembled to form a square?

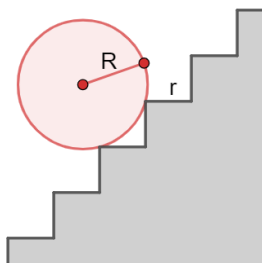
Problem 8. A mail man has six rectangular envelopes with side lengths that are integers between 1 and 10. Show that there is some envelope that can cover another of the envelopes.

Problem 9. A convex hexagon $ABCDEF$ is given, where opposite sides are parallel and of equal length (i.e. $AB = DE$, $BC = EF$ and $CD = FA$). Show that the three diagonals AD , BE and CF intersect in a single point.



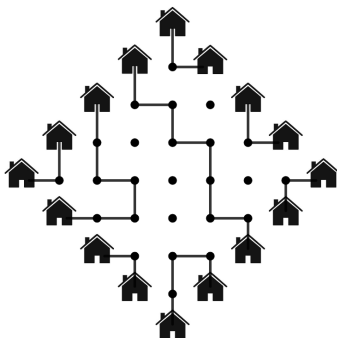
Problem 9

Problem 10. A wheel of radius R is rolling up a staircase whose steps are squares of side length $r < R$. It is given that $R^2 = 2r^2$. How many steps does the wheel need to roll up before it has rotated a full revolution?



Problem 10

Problem 11. In a town, sixteen houses are arranged in the shape of a diamond (see image). Each house is connected to exactly one other house via a path. The paths follow the grid and cannot split or cross other paths. Show that regardless of how the houses are paired and how the paths are laid out, there is at least one point in the grid that is not crossed by any path.



Problem 11