Tyngdpunkten

Harder, English

November 2024

Every problem can give at most 10 points. Good luck!

1. (Agent 007)

In the James Bond movie *Moonraker* from 1979, the agent Bond is placed in a centrifuge by the bad guys. James bond is sat in a tiny cabin at the end of an arm that is rotating in the horizontal plane around a fix axis. UVS Fysiker have studied the scene with high precision. We concluded that the period of the centrifuge was 1.5 s and the length of the arm was 10 m. Determine the centripetal acceleration the poor agent is experiencing while he is spinning. How many times larger than the gravitational acceleration g is this centripetal acceleration?

2. (Two balls)

Exactly the same amount of thermal energy is added to two identical metal balls. One of the balls has been placed on a table, while the other one hangs from a cord in the ceiling. Assume that no heat is transferred between the balls and their surroundings (nor between the balls and the table or the cord). Which ball will have a higher temperature? Justify your answer.

3. (Your twin on the Moon)

You have a twin who lives on the moon. Both of you kick a football, with the same starting speed, and the same angle to the ground. How many times longer will your twin's ball fly (horizontally) before it hits the ground when compared to your own ball? Omit air resistance and assume that the moon is a perfect sphere!

Radius	1.74×10^6 m
Mass	7.35×10^{22} kg

Table 1: Some facts about the moon.

4. (Light refraction)

You want to measure the refractive index of water. To do this, you fill a large bucket (with flat bottom) with water of depth $d = 20.0$ cm. After that you pick a point P at the surface of the water. You point a laser at the point P, so that the laser beam has the angle of incidence α (see Figure 1). Then you check where the laser hits the bottom of the bucket. Finally you measure the distance x between the point where the laser hits the bottom of the bucket and the point on the bottom of the bucket located straight under P (in other words: the point that the laser beam would hit if the angle of incidence were $\alpha = 0$). Your measurements can be found in Table 2.

Task: Using the given data points, determine a value for the refractive index of water. You may assume that air has a refractive index of $n_{air} = 1$.

Remark: For maximal points it is required that *all* of the information in the table is used, preferably in the form of a graph. Only a part of the points can be obtained if only one data point is used. An error estimation is *not* required.

Table 2: Measured values of the distance x at different angles of incidence α.

α [degrees]	x [cm]
0	0
10	2.4
20	5.0
30	8.7
40	10.6
50	15.0
60	17.5

Hint: How light changes direction when it passes from a medium with refractive index n_1 to a medium with refractive index n_2 is described by Snell's law. The law states that $n_1 \sin(\alpha) = n_2 \sin(\beta)$, where α is the angle of incidence and β is the angle of refraction (see Figure 1).

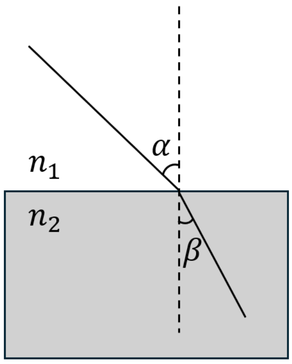

Figure 1: The image shows a beam that refracts when it goes from a medium with refractive index n_1 to a medium with refractive index n_2. The angle of incidence α is related to the angle of refraction β by Snell's law: $n_1 \sin(\alpha) = n_2 \sin(\beta)$.

5. (Coefficient of friction)

Determine experimentally the coefficient of friction μ between two papers. Describe in detail how you proceed.

Permitted tools: Ruler, paper (for example exam paper), calculator. You are especially *not* permitted to use a dynamometer.

Remark: In this assignment your method of solving the problem is primarily assessed, and not your numerical answer. Although a completed measurement and a numerical value is required for full points!

6. (Four resistors)

You have four resistors with the resistances $10\,\Omega$, $20\,\Omega$, $30\,\Omega$ and $40\,\Omega$. You also have a voltage source with EMF (Electromotive Force) $\mathcal{E} = 20\,\text{V}$ and an internal resistance of $r = 25\,\Omega$. How should you connect the resistors to the voltage source so that the power that is developed in the resistors is maximal? You need to use every resistor exactly once. Justify your answer.

7. (A magnetic field and a loop)

You have a closed, circular loop with radius r, made out of a metal wire. The wire has resistance R. A magnetic field runs perpendicular to the loop, whose magnetic flux density B varies with time t according to $B(t) = B_0 + B_1 t + B_2 t^2$, where B_0, B_1 and B_2 are constants (with differing units of course). Determine an expression for how the current I in the loop relates to time, i.e. find $I(t)$.

Remark: You do not need to care about the direction of the current. Omit the loop's self inductance.

8. (A bow and an arrow)

An arrow is shot straight up by a bow. The arrow has mass $m = 20\,\mathrm{g}$ and the bowstring has length $l = 1.0\,\mathrm{m}$. The arrow is pulled back $h_0 = 5.0\,\mathrm{cm}$ (in the middle of the bowstring). Assume that the tension in the bowstring is constant, equal to $F = 250\,\mathrm{N}$. How high will the arrow fly?

Hint: use the approximation $\sin(\alpha) \approx \tan(\alpha) \approx \alpha$, which is valid for small angles α (when α is measured in radians).

9. (Melting snow)

In this excercise we will study heat conduction. Throughout the entire excercise we will assume the temperature in every point to be *constant* in time.

Description of the setup: Consider a roof made of concrete (see the Figure 2). On top of the roof there is a flat, homogeneous snow layer. The roof has thickness $\Delta x_r = 20\,\mathrm{cm}$ and is made out of concrete with thermal conductivity (see the explanation below) $\lambda_r = 1.1\,\mathrm{W/(m\cdot K)}$. The snow has thermal conductivity $\lambda_s = 0.15\,\mathrm{W/(m\cdot K)}$. The temperature on the inside of the roof is constant with the value $T_i = 15\,^\circ\mathrm{C}$ and the temperature outside is constant with the value $T_o = -20\,^\circ\mathrm{C}$.

Question: Determine the maximal thickness of the snow layer so that the snow which is in contact with the roof does not melt.

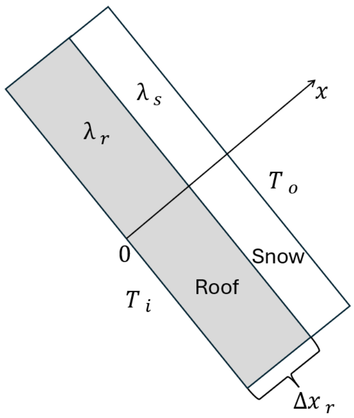

Figure 2: The figure shows the setup.

Theory: You will need Fourier's law, which we now explain (in the context of this excercise). Consider a part of the roof with area A. Define an x axis through the roof and the snow, perpendicularly to the roof, with $x = 0$ at the inside of the roof (see Figure 2). Let $\Phi(x)$ denote the amount of heat that flows through the cross section of the roof/snow (with area A) that is located at a distance x from the inside of the roof. The units of Φ are thus J/s. Fourier's law states that

$$\Phi(x) = -A\lambda\frac{dT}{dx},$$

where λ is the thermal conductivity (at the considered x coordinate). The minus sign comes from the fact $\Phi > 0$ (i.e. heat flows in the x direction) when $\frac{dT}{dx} < 0$ (i.e. the temperature decreases in the x direction).

Hint 1: What does it mean for $\Phi(x)$ that the temperature does not vary with time?

Hint 2: Try to make a graph of the temperature T as a function of x. What conclusions can you draw by looking at this graph?

10. (The barometric formula)

You have certainly heard that the atmospheric pressure depends on the altitude. Find an expression $p(h)$ for the pressure p at the height h above sea level. The pressure at the sea level is p_0. If you succeed, you will have derived the *Barometric formula*.

You can assume that the temperature is independent of the altitude and that air behaves as an *ideal gas*. The molar mass of air is M.

Hint: Consider a small air cube at some height h above the sea level. Write down the condition for equillibrum of forces for this air cube.

Good luck!